OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4752

Concepts for Advanced Mathematics (C2)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Insert for Question 12 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes

* 4 7 5 2 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Question **12**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

1 Find $\int \left(x - \frac{3}{x^2}\right) dx$. **[3]**

2 A sequence begins

$$1 \quad 3 \quad 5 \quad 3 \quad 1 \quad 3 \quad 5 \quad 3 \quad 1 \quad 3 \quad \ldots$$

and continues in this pattern.

(i) Find the 55th term of this sequence, showing your method. **[1]**

(ii) Find the sum of the first 55 terms of the sequence. **[2]**

3 You are given that $\sin\theta = \frac{\sqrt{2}}{3}$ and that θ is an acute angle. Find the **exact** value of $\tan\theta$. **[3]**

4 A sector of a circle has area $8.45\,\text{cm}^2$ and sector angle 0.4 radians. Calculate the radius of the sector. **[3]**

5

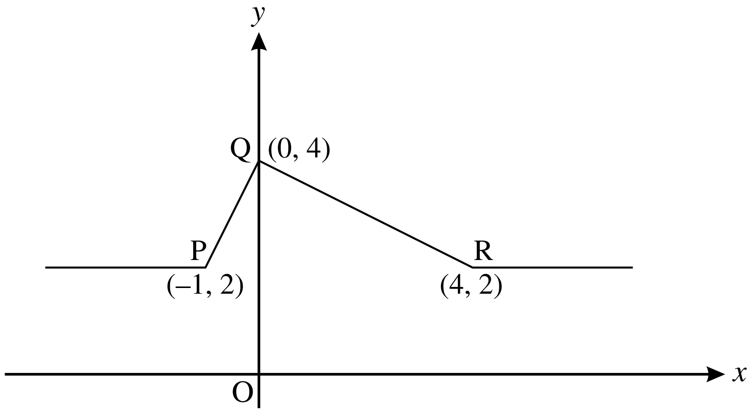

Fig. 5

Fig. 5 shows a sketch of the graph of $y = \mathrm{f}(x)$. On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to P, Q and R.

(i) $y = \mathrm{f}(2x)$ **[2]**

(ii) $y = \frac{1}{4}\mathrm{f}(x)$ **[2]**

6 **(i)** Find the 51st term of the sequence given by

$$u_1 = 5,$$
$$u_{n+1} = u_n + 4. \quad \textbf{[3]}$$

(ii) Find the sum to infinity of the geometric progression which begins

$$5 \qquad 2 \qquad 0.8 \qquad \ldots \,. \quad \textbf{[2]}$$

7

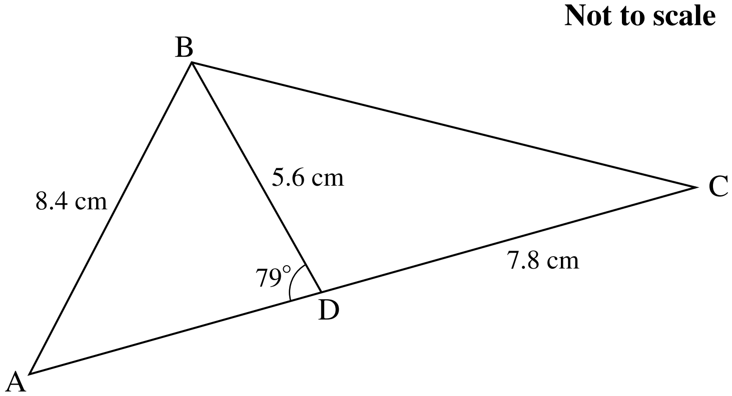

Fig. 7

Fig. 7 shows triangle ABC, with AB = 8.4 cm. D is a point on AC such that angle ADB = 79°, BD = 5.6 cm and CD = 7.8 cm.

Calculate

(i) angle BAD, **[2]**

(ii) the length BC. **[3]**

8 Find the equation of the tangent to the curve $y = 6\sqrt{x}$ at the point where $x = 16$. **[5]**

9 **(i)** Sketch the graph of $y = 3^x$. **[2]**

(ii) Use logarithms to solve $3^{2x+1} = 10$, giving your answer correct to 2 decimal places. **[3]**

Section B (36 marks)

10 **(i)** Differentiate $x^3 - 3x^2 - 9x$. Hence find the x-coordinates of the stationary points on the curve $y = x^3 - 3x^2 - 9x$, showing which is the maximum and which the minimum. **[6]**

(ii) Find, in exact form, the coordinates of the points at which the curve crosses the x-axis. **[3]**

(iii) Sketch the curve. **[2]**

11 Fig. 11 shows the cross-section of a school hall, with measurements of the height in metres taken at 1.5 m intervals from O.

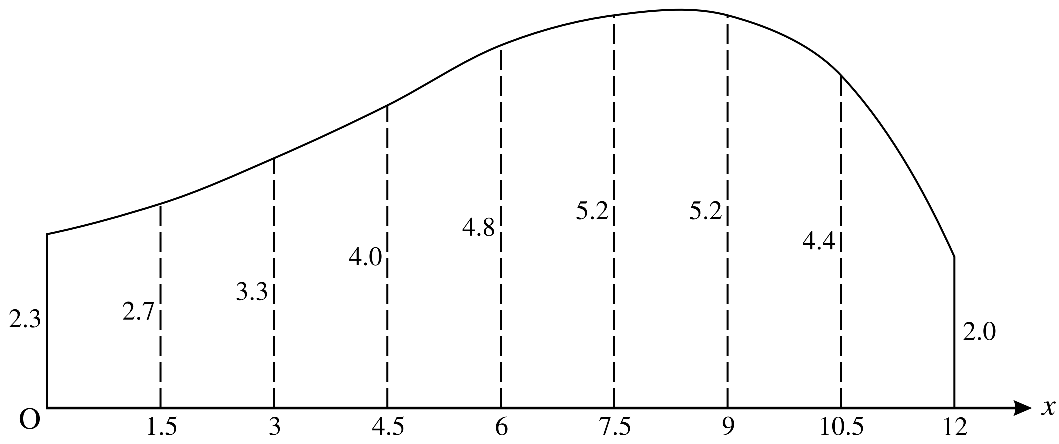

Fig. 11

(i) Use the trapezium rule with 8 strips to calculate an estimate of the area of the cross-section. **[4]**

(ii) Use 8 rectangles to calculate a lower bound for the area of the cross-section. **[2]**

The curve of the roof may be modelled by $y = -0.013x^3 + 0.16x^2 - 0.082x + 2.4$, where x metres is the horizontal distance from O across the hall, and y metres is the height.

(iii) Use integration to find the area of the cross-section according to this model. **[4]**

(iv) Comment on the accuracy of this model for the height of the hall when $x = 7.5$. **[2]**

12 Answer part (ii) of this question on the insert provided.

Since 1945 the populations of many countries have been growing. The table shows the estimated population of 15- to 59-year-olds in Africa during the period 1955 to 2005.

Year	1955	1965	1975	1985	1995	2005
Population (millions)	131	161	209	277	372	492

Source: United Nations

Such estimates are used to model future population growth and world needs of resources. One model is $P = a10^{bt}$, where the population is P millions, t is the number of years after 1945 and a and b are constants.

(i) Show that, using this model, the graph of $\log_{10} P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis. **[3]**

(ii) **On the insert**, complete the table, giving values correct to 2 decimal places, and plot the graph of $\log_{10} P$ against t. Draw, by eye, a line of best fit on your graph. **[3]**

(iii) Use your graph to find the equation for P in terms of t. **[4]**

(iv) Use your results to estimate the population of 15- to 59-year-olds in Africa in 2050. Comment, with a reason, on the reliability of this estimate. **[3]**

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OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4752

Concepts for Advanced Mathematics (C2)

INSERT for Question 12

Friday 15 January 2010

Afternoon

Duration: 1 hour 30 minutes

Candidate Forename		Candidate Surname	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

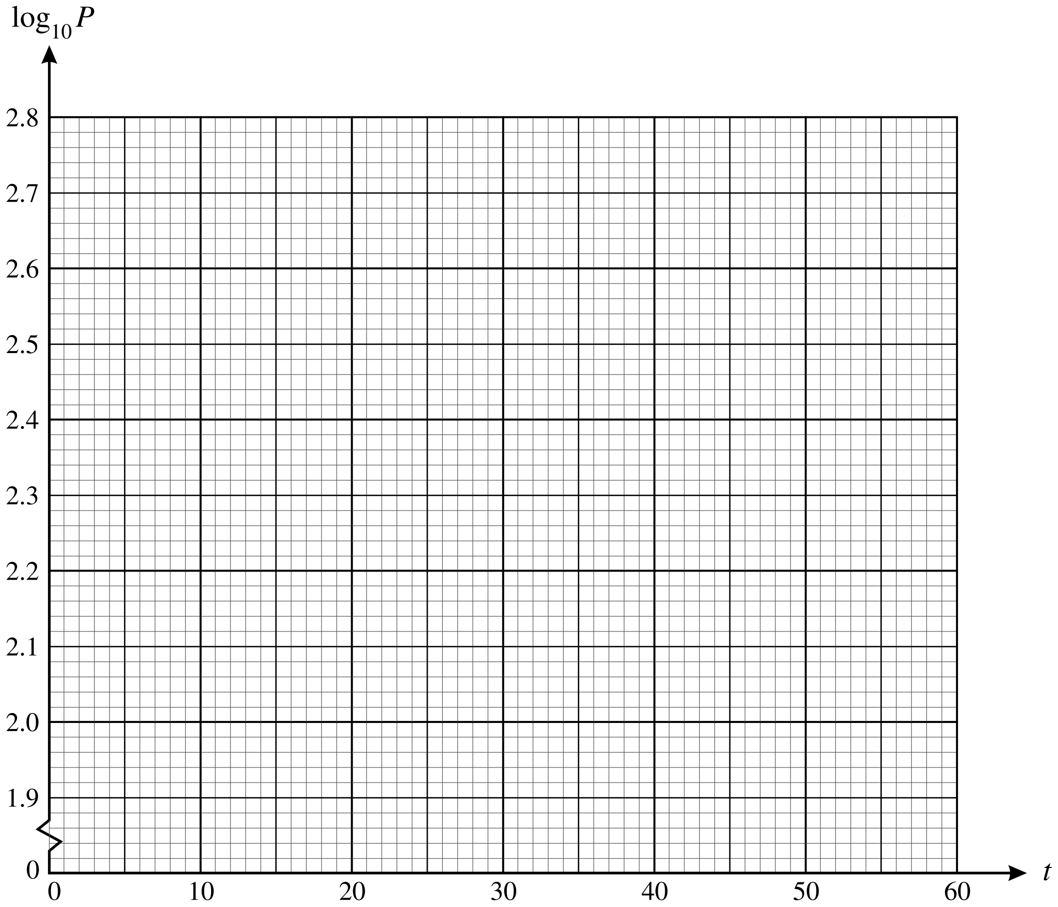
- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Question **12** part **(ii)**.
- Write your answers to Question **12** part **(ii)** in the spaces provided in this insert, and **attach it to your Answer Booklet**.

INFORMATION FOR CANDIDATES

- This document consists of **2** pages. Any blank pages are indicated.

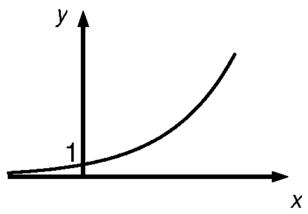
12 (ii)

Year	1955	1965	1975	1985	1995	2005
t	10	20	30	40	50	60
P	131	161	209	277	372	492
$\log_{10} P$	2.12	2.21				



4752 (C2) Concepts for Advanced Mathematics

1		$\frac{1}{2}x^2 + 3x^{-1} + c$ o.e.	3	1 for each term	3
2	**(i)** **(ii)**	5 with valid method 165 www	1 2	eg sequence has period of 4 nos. M1 for 13× (1 + 3 + 5 + 3) + 1 + 3 +5 or for 14 × (1 + 3 + 5 + 3) − 3	3
3		rt angled triangle with $\sqrt{2}$ on one side and 3 on hyp Pythag. used to obtain remaining side $= \sqrt{7}$ $\tan\theta = \frac{opp}{adj} = \frac{\sqrt{2}}{\sqrt{7}}$ o.e.	1 1 1	or M1 for $\cos^2\theta = 1 - \sin^2\theta$ used A1 for $\cos\theta = \frac{\sqrt{7}}{\sqrt{9}}$ A1 for $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{2}}{\sqrt{7}}$ o.e.	3
4		radius = 6.5 [cm]	3	M1 for $\frac{1}{2} \times r^2 \times 0.4$ [= 8.45] o.e. and M1 for $r^2 = \frac{169}{4}$ o.e. [= 42.25]	3
5	**(i)** **(ii)**	sketch of correct shape with P (−0.5,2) Q (0,4) and R (2,2) sketch of correct shape with P (−1,0.5) Q (0,1) and R (4,0.5)	2 2	1 if Q and one other are correct 1 if Q and one other are correct	4
6	**(i)** **(ii)**	205 $\frac{25}{3}$ o.e.	3 2	M1 for AP identified with $d = 4$ and M1 for 5 + 50 d used M1 for $r = \frac{2}{5}$ o.e.	5
7	**(i)** **(ii)**	$\frac{\sin A}{5.6} = \frac{\sin 79}{8.4}$ s.o.i. [A =] 40.87 to 41 [BC^2 =] $5.6^2 + 7.8^2 - 2 \times 5.6 \times 7.8 \times$ cos (“180-79”) = 108.8 to 108.9 [BC =] 10.4(…)	M1 A1 M1 A1 A1		5
8		$y' = 3x^{-\frac{1}{2}}$ ¾ when $x = 16$ $y = 24$ when $x = 16$ y − their 24 = their ¾ (x − 16) $y - 24 = \frac{3}{4}(x - 16)$ o.e.	M1 A1 B1 M1 A1	condone if unsimplified dependent on $\frac{dy}{dx}$ used for m	5

9	**(i)**		G1 DG1	for curve of correct shape in both quadrants must go through (0, 1) shown	
	(ii)	$2x + 1 = \frac{\log 10}{\log 3}$ o.e. [x =] 0.55	M1 A2	or M1 for $2x + 1 = \log_3 10$ A1 for other versions of 0.547…or 0.548	5
10	**(i)**	$3x^2 - 6x - 9$ use of their $y' = 0$ $x = -1$ $x = 3$ valid method for determining nature of turning point max at $x = -1$ and min at $x = 3$	M1 M1 A1 A1 M1 A1	 c.a.o.	6
	(ii)	$x(x^2 - 3x - 9)$ $\frac{3 \pm \sqrt{45}}{2}$ or $(x - \frac{3}{2})^2 = 9 + \frac{9}{4}$ $0, \frac{3}{2} \pm \frac{\sqrt{45}}{2}$ o.e.	M1 M1 A1		3
	(iii)	sketch of cubic with two turning points correct way up x-intercepts – negative, 0, positive shown	G1 DG1		2
11	**(i)**	47.625 [m^2] to 3 sf or more, with correct method shown	4	M3 for $\frac{1.5}{2} \times (2.3 + 2 + 2[2.7 + 3.3 + 4 + 4.8 + 5.2 + 5.2 + 4.4])$	4
	(ii)	43.05	2	M1 for 1.5 × (2.3+2.7+3.3+4+4.8+5.2+4.4+2)	2
	(iii)	$-0.013x^4/4 + 0.16x^3/3 - 0.082x^2/2 + 2.4x$ o.e. their integral evaluated at x = 12 (and 0) only 47.6 to 47.7	M2 M1 A1	M1 for three terms correct dep on integration attempted	4
	(iv)	5.30.. found compared with 5.2 s.o.i.	1 D1		2
12	**(i)**	$\log P = \log a + bt$ www comparison with $y = mx + c$ s.o.i. intercept = $\log_{10} a$	1 1 1	 must be with correct equation dependent on correct equation	3
	(ii)	[2.12, 2.21], 2.32, 2.44, 2.57, 2.69 plots ft ruled line of best fit	1 1 1	 Between (10, 2.08) and (10, 2.12)	3

	(iii)	$0.0100 \leq$ m < 0.0125	B2	M1 for $\frac{y-\text{step}}{x-\text{step}}$	
		a = 10^c or loga = c	B1	$1.96 \leq$ c ≤ 2.02	
		$P = 10^c \times 10^{mt}$ or 10^{mt+c}	B1	f.t. their m and a	4
	(iv)	use of $t = 105$ 1.0 – 2.0 billion approx unreliable since extrapolation o.e.	B1 B1 E1		3

4752 Concepts for Advanced Mathematics (C2)

General Comments

The paper was generally well received, with very few poor scripts. However, there were also fewer outstanding scripts than usual for a January series. Some high-scoring candidates lost marks through careless errors. A significant minority of candidates wasted time by using graph paper for an accurate plot when asked for a sketch. There were some very good scores in the second half of section A; many candidates scored full marks on questions 6, 7, 8 and 9.

Comments on Individual Questions

Section A

1 Many candidates lost marks on this question. Some candidates lost an easy mark by omitting "+ c". Weaker candidates failed to deal with $\frac{3}{x^2}$ correctly: sign errors were quite common, but some candidates gave the answer as $\frac{6}{x^3}$, in spite of obtaining the other term correctly.

2 This question defeated many candidates. Those candidates who did obtain the correct answer often presented no reasoning, in spite of a clear instruction to do so. A good number of candidates thought the period of the sequence was 5 or 10. Many candidates blindly applied the formula for arithmetic or geometric progressions in part (ii), or wrote 13.75 × 12 (which fortuitously gives the correct answer). Fully correct solutions were in the minority.

3 This question was generally well done. Some candidates obtained √11 or √5 for the third side of the triangle, or went straight for their calculators and presented all the numbers from their display, which didn't score.

4 This question was done very well indeed. Only a few candidates used $r\theta$ or $r\theta^2$; fewer still failed to manipulate the correct formula to obtain the correct answer. Even the majority of those who converted to degrees earned full marks by using their calculators effectively. A very small minority thought the angle was 0.4π radians.

5 There were many excellent responses to this question. In part (i) some candidates sketched 2f(x) or f(½x). The usual errors in part (ii) were to sketch 4f(x) or f(4x). Many candidates unnecessarily presented accurate plots on graph paper.

6 Part (i) was accessible to most. Nearly all candidates obtained the correct answer, with a small minority making arithmetical slips. Some weak candidates gave the answer as 51 + 4, or used the formula for the sum of the first 51 terms. A surprising number substituted n = 55 in otherwise correct working.
There were many excellent answers to part (ii). A few lost the accuracy mark by presenting the answer 8.3 or 8.33. Some weak candidates used r = 2.5, and did not score.

7 Most scored very well on this question. Some lost the accuracy mark in part (i) through premature rounding or truncating the final answer. A few used $\sin\theta = \frac{o}{h}$ and did not score. The majority of candidates scored full marks in part (ii), even when they used convoluted methods to get there. Some weaker candidates tried to use the Sine Rule, or used sin*θ* instead of cos*θ* in the Cosine Rule formula. Others failed to cope with the double negative.

8 The majority of candidates scored very well indeed on this question. Only a few candidates differentiated $x^{\frac{1}{6}}$, and a few slipped up by giving $\frac{dy}{dx}$ as $12x^{-\frac{1}{2}}$. Most found *y* = 24 correctly and obtained the following method mark. Some candidates used the gradient of the normal at this stage, thus losing the last two marks. A few candidates took *m* to be $3x^{-\frac{1}{2}}$.

9 Part (i) was very well done. Some candidates failed to score because they only sketched the curve in one quadrant, and a few lost a mark by failing to identify the *y*- intercept. In part (ii) most scored full marks. A few lost the final mark by presenting the answer to 3 s.f. $x = \frac{1}{2}(\frac{1}{\log 3} + 1)$ and $x = \frac{1}{2}\frac{1}{\log 3} - 1$ were occasionally seen.

Section B

10 **(i)** This was done very well indeed, with many candidates scoring full marks. A few candidates failed to justify their identification of the nature of the turning points, or used the second derivative and reached the wrong conclusion.

(ii) This was not done well. Some candidates went straight to the quadratic formula and consequently missed the value *x* = 0 and did not earn any marks. Most simply earned the first mark because they only presented decimal expansions of the two irrational roots instead of giving them in surd form.

(iii) Most candidates were able to score both marks, even if they had not been entirely successful in parts (i) and (ii).

11 **(i)** The majority of candidates scored full marks here. Most went straight to the composite form of the Trapezium Rule. A few candidates made arithmetic slips, thus losing the accuracy mark, and a small number lost all the marks by omitting the outside brackets or by substituting *h* = 12. A few candidates wasted time by calculating the area of each individual trapezium before summing.

(ii) Many candidates simply used the lower x value throughout, thus failing to score.

(iii) This was generally well done. A common slip was integrating 2.4 to obtain $24x$, and many candidates lost the final accuracy mark through poor calculator work. Some lost the last two marks because incorrect limits were used.

(iv) A number of candidates simply made a general comment here, which didn't score. Others substituted in their integrated function. Those who did obtain the correct value of 5.3… did not always appreciate the need to compare this with 5.2.

12 **(i)** Far too many candidates lost marks on this standard piece of bookwork, either through failing to produce a correct formula, or through presenting the correct formula after wrong working. Many candidates left $\log_{10} 10$ unsimplified, which was penalised.

(ii) The overwhelming majority of candidates scored full marks on this section. However, 2.36 was sometimes seen instead of 2.44, and lines were not always ruled, which cost a mark.

(iii) The expected approach of log*a* = intercept and *b* = gradient generally yielded the first three marks, although some failed to obtain a gradient within the required range. A few candidates were not sure whether “a” was c or 10^c. Only the best candidates then went on to present the equation in the required form.

(iv) *t* = 105 was often used appropriately, but *t* = 95, 2050, 105 and 110 were also seen. A good number lost an easy mark by presenting an ambiguous answer – it needed to be clear that the candidate was talking about thousands of millions of people, not thousands of people, to score. The final mark was rarely awarded. Many commented on natural disasters rather than commenting directly or indirectly on the problems associated with extrapolation well beyond the range of available data.